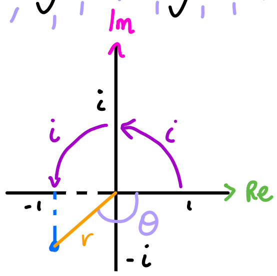
# Complex Numbers

## Argand Diagram




$z = x + iy$ (real part $x$, imaginary part $y$)

$i = \sqrt{-1}$

- real terms grouped → $Re(z) = Re(\bar{z})$
- imaginary terms grouped → $-Im(z) = Im(\bar{z})$

Imaginary part does not include 'i'

change sign for conjugate $\bar{z}$

$z + \bar{z} = 2Re(z)$

$z - \bar{z} = 2i\,Im(z)$

$\bar{\bar{z}} = z$

$\bar{z^n} = \bar{z}^n$

$|z|^2 = z\bar{z}$

Conjugate - reflect in real axis

### Polar form

$x + iy \rightarrow r\cos\theta + i\,r\sin\theta$

$= r(\cos\theta + i\sin\theta)$

argument (arg.)

$-\pi < \theta \leq \pi$ aka. $\theta \in (-\pi, \pi]$

$= \arg z. = \tan^{-1}\left(\frac{Im}{Re}\right)$

## Exponential form

$re^{i\theta}$

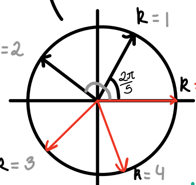
$e^{i\theta} = \cos\theta + i\sin\theta$ — Euler's FORMULA

$1 = e^{i2n\pi}$

$\cos\theta = Re(e^{i\theta}) = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$\sin\theta = Im(e^{i\theta}) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

## Solving powers equations




each root equally spaced

$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{i\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)}$

$$\boxed{z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)\right]}$$

$k = 0, 1, 2 \ldots n-1$

$$\boxed{z^{p} = r^{p}\left[\cos(p\theta + 2\pi k p) + i\sin(p\theta + 2\pi k p)\right]}$$

where $p = \frac{m}{n}$

## Multiplying

expand as normal

remember $i^2 = -1$, $i^4 = 1$ etc.

$r_1 r_2\left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$

easier in polar form

## Dividing

Multiply top and bottom by conjugate of denomintator

$\frac{z_1\bar{z}_2}{z_2\bar{z}_2}$ 'removed'

$\frac{r_1}{r_2}\underbrace{\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]}_{e^{i(\theta_1 - \theta_2)}}$

## Powers

De Moivre's THEOREM

$\cos(n\theta) + i\sin(n\theta) = [\cos(\theta) + i\sin(\theta)]^n$

$r^n e^{in\theta}$

to multiple angles

### Of trig. functions

$z^n = \cos n\theta + i\sin n\theta$

$$\therefore \boxed{\begin{aligned} z^n + z^{-n} &= 2\cos n\theta \\ z^n - z^{-n} &= 2i\sin n\theta \end{aligned}}$$

## Roots

If polynomial has only real coefficients

roots occur in conjugate pairs

e.g if cubic only has real coefficients

3 real roots

or

2 conjugates and 1 real